CLASSIFICATIONS DES ALGBRES DE LIE NILPOTENTES COMPLEXES DE DIMENSION 7 OU MOINS

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1. Introduction

On ne propose que la classification, à isomorphisme près, des algèbres de Lie nilpotentes complexes ind'ecomposables, c'est-à-dire qui ne sont pas somme directe d'idéaux non triviaux. En effet ces dernières se séduisent aisément de la liste des indécomposables. Rappelons également que les algèbres de Lie sont décritent par la donnée du crochet de Lie dans une base fixée. Seuls les crochets non nuls et qui ne se déduisent pas par la propriété d'antisymétrie sont écrits.

Cette classification ne concerne que les algèbres de Lie de dimension inférieure ou égale à 7, au delà iol n'y a pas au jour d'aujourd'hui de liste complètement établie.

2. Dimension $1, 2, 3$

— En dimension 1, il n'existe que l'algèbre abélienne.
— En dimension 2, toute algèbre de Lie est décomposable et abélienne.
— En dimension 3, toute algèbre de Lie nilpotente est isomorphe à l'algèbre de Heisenberg $\mathfrak{h}_1 = \mathfrak{n}_3^1$ qui est filiforme et métabélienne
$\mathfrak{n}_3^1 = \mathfrak{h}_3 : [X_1, X_2] = X_3$.

3. Dimension 4

$\mathfrak{n}_4^1 : [X_1, X_i] = X_{i+1},\ i = 2, 3.$ (filiforme)

4. Dimension 5

— $c(\mathfrak{g}) = (4, 1)$
$\mathfrak{n}_5^1 : [X_1, X_i] = X_{i+1},\ i = 2, 3, 4,\ [X_2, X_3] = X_5.$
$\mathfrak{n}_5^2 : [X_1, X_i] = X_{i+1},\ i = 2, 3, 4.$
— $c(\mathfrak{g}) = (3, 1, 1)$
$\mathfrak{n}_5^3 : [X_1, X_i] = X_{i+1},\ i = 2, 3\ [X_2, X_5] = X_4.$
$\mathfrak{n}_5^4 : [X_1, X_i] = X_{i+1},\ i = 2, 3,\ [X_2, X_3] = X_5.$
— $c(\mathfrak{g}) = (2, 2, 1)$
$\mathfrak{n}_5^5 : [X_1, X_2] = X_3\ [X_1, X_4] = X_5.$
— $c(\mathfrak{g}) = (2, 1, 1, 1)$
$\mathfrak{n}_5^6 : [X_1, X_2] = X_3\ [X_4, X_5] = X_3.$ l'algèbre de Heisenberg $\mathfrak{h}_2$.

Date: Chevat 16, 5775.

5. Dimension 6

— $c(\mathfrak{g}) = (5,1)$

$\mathfrak{n}_6^1 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,5\ \ [X_2, X_3] = X_5,\ [X_2, X_4] = X_6.$

$\mathfrak{n}_6^2 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,5\ \ [X_2, X_3] = X_6.$

$\mathfrak{n}_6^3 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,5\ \ [X_2, X_5] = X_6,\ [X_3, X_4] = -X_6.$

$\mathfrak{n}_6^4 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,5\ \ [X_2, X_3] = X_5 - X_6,\ [X_2, X_4] = X_6,\ [X_2, X_5] = X_6,\ [X_3, X_4] = -X_6.$

$\mathfrak{n}_6^5 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,5.$

— $c(\mathfrak{g}) = (4,1,1)$

$\mathfrak{n}_6^6 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,\ [X_2, X_3] = X_5\ \ [X_2, X_6] = X_5.$

$\mathfrak{n}_6^7 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,\ [X_2, X_6] = X_4\ \ [X_3, X_6] = X_5.$

$\mathfrak{n}_6^8 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,\ [X_2, X_6] = X_5$

$\mathfrak{n}_6^9 : [X_1, X_i] = X_{i+1},\ i = 2,3,4,\ [X_2, X_3] = X_6$

$\mathfrak{n}_6^{10} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_2, X_3] = X_6\ \ [X_2, X_6] = X_5.$

— $c(\mathfrak{g}) = (3,2,1)$

$\mathfrak{n}_6^{11} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_5, X_6] = X_4$

$\mathfrak{n}_6^{12} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_2, X_5] = X_4$

$\mathfrak{n}_6^{13} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_2, X_3] = X_6\ \ [X_2, X_5] = X_6$

$\mathfrak{n}_6^{14} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_2, X_3] = X_4 - X_6\ \ [X_2, X_5] = X_6$

$\mathfrak{n}_6^{15} : [X_1, X_i] = X_{i+1},\ i = 2,3,5,\ [X_2, X_5] = X_6\ \ [X_5, X_6] = X_4$

$\mathfrak{n}_6^{16} : [X_1, X_i] = X_{i+1},\ i = 2,3,5\ \ [X_2, X_3] = X_4$

$\mathfrak{n}_6^{17} : [X_1, X_i] = X_{i+1},\ i = 2,3,5$

— $c(\mathfrak{g}) = (3,1,1,1)$

$\mathfrak{n}_6^{18} : [X_1, X_i] = X_{i+1},\ i = 2,3,\ [X_5, X_6] = X_4$

— $c(\mathfrak{g}) = (2,2,1,1)$

$\mathfrak{n}_6^{19} : [X_1, X_i] = X_{i+1},\ i = 2,4,\ [X_2, X_6] = X_5$

$\mathfrak{n}_6^{20} : [X_1, X_i] = X_{i+1},\ i = 2,4,\ [X_2, X_4] = X_6$

6. Dimension 7

— $c(\mathfrak{g})) = (6,1)$, cas filiforme.

$$\mathfrak{n}_7^1(\alpha) : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = (1+\alpha)X_5,\ [X_2, X_4] = (1+\alpha)X_6, \\ [X_2, X_5] = \alpha X_7,\ [X_3, X_4] = X_7. \end{cases}$$

$\mathfrak{n}_7^2 : \ [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = X_5,\ [X_2, X_4] = X_6,\ [X_2, X_5] = X_7.$

$\mathfrak{n}_7^3 : \ [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = X_5 + X_7,\ [X_2, X_4] = X_6,\ [X_2, X_5] = X_7.$

$\mathfrak{n}_7^4 : \ [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = X_6,\ [X_2, X_4] = X_7,\ [X_2, X_5] = X_7,\ [X_3, X_4] = -X_7.$

$\mathfrak{n}_7^5 : [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = X_6 + X_7,\ [X_2, X_4] = X_7.$

$\mathfrak{n}_7^6 : [X_1, X_i] = X_{i+1}, \quad 2 \le i \le 6,\ [X_2, X_3] = X_6,\ [X_2, X_4] = X_7.$

$\mathfrak{n}_7^7 : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 6,\ [X_2, X_3] = X_7.$

$\mathfrak{n}_7^8 : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 6.$

— $c(\mathfrak{g}) = (5, 1, 1)$.

$$\mathfrak{n}_7^9 : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_5,\ [X_2, X_4] = X_6, \\ [X_2, X_5] = X_7,\ [X_3, X_4] = -X_7 \end{cases}$$

$\mathfrak{n}_7^{10} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_6,\ [X_2, X_5] = X_7,\ [X_3, X_4] = -X_7.$

$\mathfrak{n}_7^{11} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_5] = X_7,\ [X_3, X_4] = -X_7.$

$$\mathfrak{n}_7^{12}(\alpha) : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_5 + X_7,\ [X_2, X_4] = X_6, \\ [X_2, X_7] = X_5 + \alpha X_6,\ [X_3, X_7] = X_6 \end{cases}$$

$\mathfrak{n}_7^{13} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_7,\ [X_2, X_7] = X_5 - X_6,\ [X_3, X_7] = X_6.$

$$\mathfrak{n}_7^{14} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_4 - X_7,\ [X_2, X_4] = X_5, \\ [X_2, X_7] = X_5 - 2X_6,\ [X_3, X_4] = X_6,\ [X_3, X_7] = X_6. \end{cases}$$

$$\mathfrak{n}_7^{15} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = \frac{1}{2}X_4 + \frac{1}{4}X_5 - \frac{1}{8}X_6 - \frac{1}{2}X_7,\ [X_2, X_4] = \frac{1}{2}X_5 + \frac{1}{4}X_6, \\ [X_2, X_5] = X_6,\ [X_2, X_7] = \frac{1}{2}X_5 - \frac{1}{4}X_6,\ [X_3, X_4] = -\frac{1}{2}X_6,\ [X_3, X_7] = \frac{1}{2}X_6. \end{cases}$$

$$\mathfrak{n}_7^{16} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = \frac{1}{2}X_4 + \frac{1}{4}X_5 - \frac{1}{8}X_6 - \frac{1}{2}X_7,\ [X_2, X_4] = \frac{1}{2}X_5 + \frac{1}{4}X_6, \\ [X_2, X_7] = \frac{1}{2}X_5 - \frac{3}{4}X_6,\ [X_3, X_4] = \frac{1}{2}X_6,\ [X_3, X_7] = \frac{1}{2}X_6. \end{cases}$$

$$\mathfrak{n}_7^{17} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = -X_4 + X_7,\ [X_2, X_4] = -X_5, \\ [X_2, X_7] = X_5,\ [X_3, X_4] = -X_6,\ [X_3, X_7] = X_6. \end{cases}$$

$$\mathfrak{n}_7^{18} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = \frac{1}{2}X_4 + \frac{1}{2}X_7,\ [X_2, X_4] = \frac{1}{2}X_5, \\ [X_2, X_7] = -\frac{1}{2}X_5,\ [X_3, X_4] = \frac{1}{2}X_6,\ [X_3, X_7] = -\frac{1}{2}X_6. \end{cases}$$

$$\mathfrak{n}_7^{19} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_4 - 2X_5 + X_7,\ [X_2, X_4] = X_5 - 2X_6, \\ [X_2, X_7] = X_6,\ [X_3, X_4] = X_6. \end{cases}$$

$$\mathfrak{n}_7^{20} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_4 + X_5 + X_7,\ [X_2, X_4] = X_5 + X_6, \\ [X_2, X_7] = X_6,\ [X_3, X_4] = X_6. \end{cases}$$

$\mathfrak{n}_7^{21} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_7,\ [X_2, X_7] = X_6.$

$$\mathfrak{n}_7^{22} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_4 + X_5 + X_7,\ [X_2, X_4] = X_5 + X_6, \\ [X_3, X_4] = X_6. \end{cases}$$

$\mathfrak{n}_7^{23} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_4 + X_7,\ [X_2, X_4] = X_5,\ [X_3, X_4] = X_6.$

$\mathfrak{n}_7^{24} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_5 + X_7,\ [X_2, X_4] = X_6.$

$\mathfrak{n}_7^{25} : [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_3] = X_7.$

$$\mathfrak{n}_7^{26} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_3, X_7] = X_6,\ [X_2, X_7] = X_5, \\ [X_3, X_4] = X_6,\ [X_2, X_4] = X_5 + X_6,\ [X_2, X_3] = X_4 + X_5. \end{cases}$$

$$\mathfrak{n}_7^{27} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_4] = X_5,\ [X_2, X_3] = X_4, \\ [X_3, X_4] = X_6,\ [X_2, X_7] = X_5 + X_6,\ [X_3, X_7] = X_6. \end{cases}$$

$$\mathfrak{n}_7^{28} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_4] = X_5,\ [X_2, X_3] = X_4, \\ [X_3, X_4] = X_6,\ [X_2, X_7] = X_5,\ [X_3, X_7] = X_6. \end{cases}$$

$$\mathfrak{n}_7^{29} : \begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 5,\ [X_2, X_7] = X_6,\ [X_3, X_4] = X_6, \\ [X_2, X_4] = X_5 + X_6,\ [X_2, X_3] = X_4 + X_5 \end{cases}$$

$$\mathfrak{n}_7^{30}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_3,X_4]=X_6,\ [X_2,X_4]=X_5+X_6,\\ [X_2,X_3]=X_4+X_5,\ [X_2,X_7]=X_5+X_6,\ [X_3,X_7]=X_6. \end{cases}$$

$$\mathfrak{n}_7^{31}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_2,X_7]=X_6,\ [X_3,X_4]=X_6,\\ [X_2,X_4]=X_5,\ [X_2,X_3]=X_4. \end{cases}$$

$$\mathfrak{n}_7^{32}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_4,X_7]=X_6,\ [X_3,X_7]=X_5,\\ [X_2,X_7]=X_4,\ [X_2,X_3]=X_6. \end{cases}$$

$$\mathfrak{n}_7^{33}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_4,X_7]=X_6,\ [X_3,X_7]=X_5,\ [X_2,X_7]=X_4+X_6.$$

$$\mathfrak{n}_7^{34}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_4,X_7]=X_6,\ [X_3,X_7]=X_5,\ [X_2,X_7]=X_4.$$

$$\mathfrak{n}_7^{35}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_3,X_7]=X_6,\ [X_2,X_7]=X_5,\\ [X_2,X_4]=X_6,\ [X_2,X_3]=X_5. \end{cases}$$

$$\mathfrak{n}_7^{36}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_3,X_7]=X_6,\ [X_2,X_7]=X_5+X_6.$$

$$\mathfrak{n}_7^{37}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_3,X_7]=X_6,\ [X_2,X_7]=X_5.$$

$$\mathfrak{n}_7^{38}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_2,X_7]=X_6,\ [X_2,X_4]=X_6,\ [X_2,X_3]=X_5.$$

$$\mathfrak{n}_7^{39}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_2,X_7]=X_6,\ [X_2,X_3]=X_6.$$

$$\mathfrak{n}_7^{40}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 5,\ [X_2,X_7]=X_6.$$

— $c(\mathfrak{g})=(4,1,1,1)$.

$$\mathfrak{n}_7^{41}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_6,X_7]=X_5,\ [X_3,X_7]=X_5,\\ [X_2,X_7]=X_4,\ [X_2,X_3]=X_5. \end{cases}$$

$$\mathfrak{n}_7^{42}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_3,X_7]=X_5,\ [X_2,X_7]=X_4,\ [X_2,X_6]=X_5.$$

$$\mathfrak{n}_7^{43}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_6,X_7]=X_5,\ [X_3,X_7]=X_5,\ [X_2,X_7]=X_4.$$

$$\mathfrak{n}_7^{44}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_2,X_6]=X_5,\ [X_2,X_3]=X_7.$$

$$\mathfrak{n}_7^{45}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_6,X_7]=X_5,\ [X_2,X_3]=X_5.$$

$$\mathfrak{n}_7^{46}: [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_6,X_7]=X_5.$$

— $c(\mathfrak{g})=(4,2,1)$.

$$\mathfrak{n}_7^{47}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_4]=X_7,\\ [X_2,X_3]=X_6,\ [X_2,X_6]=X_5. \end{cases}$$

$$\mathfrak{n}_7^{48}: \ [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_4]=X_7,\ [X_2,X_3]=X_6.$$

$$\mathfrak{n}_7^{49}(\alpha): \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_3]=X_7,\\ [X_3,X_6]=\alpha X_5,\ [X_2,X_7]=X_5,\ [X_2,X_6]=(\alpha+1)X_4. \end{cases}$$

$$\mathfrak{n}_7^{50}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_3]=X_7,\\ [X_3,X_6]=X_5,\ [X_2,X_6]=X_4. \end{cases}$$

$$\mathfrak{n}_7^{51}: \ [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_3]=X_7,\ [X_2,X_6]=X_7.$$

$$\mathfrak{n}_7^{52}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_3]=X_7,\\ [X_3,X_6]=X_5,\ [X_2,X_6]=X_7. \end{cases}$$

$$\mathfrak{n}_7^{53}: \begin{cases} [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_1,X_6]=X_7,\ [X_2,X_3]=X_4-X_7,\\ [X_2,X_4]=X_5,\ [X_2,X_7]=X_5,\ [X_2,X_6]=X_4-X_7. \end{cases}$$

$$\mathfrak{n}_7^{54}: \ [X_1,X_i]=X_{i+1}, \quad 2\leq i\leq 4,\ [X_2,X_3]=X_7,\ [X_2,X_6]=X_5+X_7.$$

$\mathfrak{n}_7^{55}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_7,\ [X_2, X_6] = X_5.$

$\mathfrak{n}_7^{56}$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_7,\ [X_3, X_6] = 3X_5, \\ [X_2, X_7] = -X_5,\ [X_2, X_6] = 2X_4 + X_7. \end{cases}$

$\mathfrak{n}_7^{57}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_7.$

$\mathfrak{n}_7^{58}$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5, \\ [X_2, X_6] = X_7,\ [X_6, X_7] = X_5. \end{cases}$

$\mathfrak{n}_7^{59}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_6] = X_7,\ [X_6, X_7] = X_5.$

$\mathfrak{n}_7^{60}(\alpha)$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = \alpha X_5,\ [X_3, X_6] = X_5 \\ [X_2, X_6] = X_4,\ [X_6, X_7] = X_5. \end{cases}$

$\mathfrak{n}_7^{61}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5,\ [X_6, X_7] = X_5.$

$\mathfrak{n}_7^{62}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_6, X_7] = X_5.$

$\mathfrak{n}_7^{63}$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_3, X_6] = -X_5, \\ [X_2, X_7] = X_5,\ [X_2, X_6] = X_7. \end{cases}$

$\mathfrak{n}_7^{64}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_3, X_6] = X_5,\ [X_2, X_6] = X_4 + X_7.$

$\mathfrak{n}_7^{65}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_7] = X_5,\ [X_2, X_6] = X_4 + X_7.$

$\mathfrak{n}_7^{66}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5,\ [X_2, X_6] = X_7.$

$\mathfrak{n}_7^{67}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_6] = X_7.$

$\mathfrak{n}_7^{68}(\alpha)$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_3, X_6] = \alpha X_5, \\ [X_2, X_7] = X_5,\ [X_2, X_6] = (1 + \alpha) X_4. \end{cases}$

$\mathfrak{n}_7^{69}$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5, \\ [X_3, X_6] = X_5,\ [X_2, X_7] = X_5,\ [X_2, X_6] = 2X_4. \end{cases}$

$\mathfrak{n}_7^{70}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_3, X_6] = X_5,\ [X_2, X_7] = -X_5.$

$\mathfrak{n}_7^{71}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_3, X_6] = X_5,\ [X_2, X_6] = X_4.$

$\mathfrak{n}_7^{72}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5,\ [X_2, X_6] = X_5.$

$\mathfrak{n}_7^{73}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = X_5.$

$\mathfrak{n}_7^{74}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_6] = X_5.$

$\mathfrak{n}_7^{75}$: $[X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7.$

$\mathfrak{n}_7^{76}$: $\begin{cases} [X_1, X_i] = X_{i+1}, \quad 2 \leq i \leq 4,\ [X_1, X_6] = X_7,\ [X_2, X_3] = -X_7, \\ [X_2, X_6] = X_4 + 2X_7,\ [X_3, X_6] = X_5. \end{cases}$

— $c(\mathfrak{g}) = (3, 3, 1)$.

$\mathfrak{n}_7^{77}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_2, X_5] = X_7.$

$\mathfrak{n}_7^{78}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_2, X_6] = X_4,\ [X_2, X_5] = X_3.$

$\mathfrak{n}_7^{79}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_5, X_6] = X_4,\ [X_2, X_5] = X_7.$

$\mathfrak{n}_7^{80}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6.$

$\mathfrak{n}_7^{81}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_5, X_6] = X_4.$

$\mathfrak{n}_7^{82}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_5, X_6] = X_4,\ [X_2, X_3] = X_7.$

$\mathfrak{n}_7^{83}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5, 6, \quad [X_5, X_6] = X_7,\ [X_2, X_3] = X_4 + X_7.$

$\mathfrak{n}_7^{84}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_7,\ [X_2, X_3] = X_4.$

$\mathfrak{n}_7^{85}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \\ [X_3, X_5] = X_7,\ [X_2, X_5] = X_4 + X_6, \\ [X_2, X_3] = X_4. \end{cases}$

$\mathfrak{n}_7^{86}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_7,\ [X_2, X_3] = X_7.$

$\mathfrak{n}_7^{87}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_7 + X_4,\ [X_2, X_6] = X_4, \\ [X_2, X_5] = X_3. \end{cases}$

$\mathfrak{n}_7^{88}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_4,\ [X_3, X_5] = X_7, \\ [X_2, X_3] = X_4,\ [X_2, X_5] = X_6. \end{cases}$

$\mathfrak{n}_7^{89}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_4,\ [X_2, X_3] = X_4, \\ [X_2, X_5] = X_7. \end{cases}$

$\mathfrak{n}_7^{90}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_2, X_3] = X_4,\ [X_3, X_5] = X_7, \\ [X_2, X_5] = X_6. \end{cases}$

$\mathfrak{n}_7^{91}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_5, X_6] = X_7.$

$\mathfrak{n}_7^{92}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5,6, \ \ [X_2, X_3] = X_4,\ [X_2, X_5] = X_7.$

— $c(\mathfrak{g}) = (3,2,1,1).$

$\mathfrak{n}_7^{93}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_5] = X_7.$

$\mathfrak{n}_7^{94}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_5] = X_4,\ [X_2, X_3] = X_7$

$\mathfrak{n}_7^{95}: \begin{array}{l} [X_1, X_i] = X_{i+1},\ i = 2,3,5 \\ [X_2, X_3] = X_7 \end{array}$

$\mathfrak{n}_7^{96}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_5] = X_7,\ [X_2, X_6] = X_4.$

$\mathfrak{n}_7^{97}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_6] = X_4,\ [X_3, X_5] = -X_4, \\ [X_2, X_5] = X_7. \end{cases}$

$\mathfrak{n}_7^{98}: \begin{cases} [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_6] = X_4,\ [X_3, X_5] = -X_4, \\ [X_2, X_5] = X_7,\ [X_5, X_6] = X_4. \end{cases}$

$\mathfrak{n}_7^{99}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_6,\ [X_2, X_3] = X_4.$

$\mathfrak{n}_7^{100}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_4,\ [X_5, X_7] = X_6.$

$\mathfrak{n}_7^{101}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_6,\ [X_5, X_7] = X_4.$

$\mathfrak{n}_7^{102}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_5, X_7] = X_4.$

$\mathfrak{n}_7^{103}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_4.$

$\mathfrak{n}_7^{104}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_5, X_7] = X_4,\ [X_2, X_3] = X_4.$

$\mathfrak{n}_7^{105}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_4,\ [X_2, X_3] = X_4.$

$\mathfrak{n}_7^{106}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_4,\ [X_5, X_6] = X_4.$

$\mathfrak{n}_7^{107}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_5, X_7] = X_3,\ [X_6, X_7] = X_4.$

$\mathfrak{n}_7^{108}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_7] = X_6,\ [X_2, X_3] = X_6.$

$\mathfrak{n}_7^{109}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_5, X_7] = X_6,\ [X_2, X_3] = X_6.$

$\mathfrak{n}_7^{110}: \ [X_1, X_i] = X_{i+1},\ i = 2,3,5, \ \ [X_2, X_3] = X_6,\ [X_5, X_7] = X_4.$

$\mathfrak{n}_7^{111}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_7] = X_6,\ [X_2, X_5] = X_4.$

$\mathfrak{n}_7^{112}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_3] = X_6,\ [X_2, X_7] = X_4.$

$\mathfrak{n}_7^{113}$: $[X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_5, X_7] = X_6,\ [X_5, X_6] = X_4.$

$\mathfrak{n}_7^{114}$: $\begin{cases} [X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_7] = X_4,\ [X_5, X_6] = X_4, \\ [X_5, X_7] = X_6. \end{cases}$

$\mathfrak{n}_7^{115}$: $\begin{cases} [X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_5] = X_4,\ [X_5, X_7] = X_3, \\ [X_6, X_7] = X_4. \end{cases}$

$\mathfrak{n}_7^{116}$: $\begin{cases} [X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_3, X_5] = -X_4,\ [X_2, X_6] = X_4, \\ [X_5, X_7] = -X_4. \end{cases}$

$\mathfrak{n}_7^{117}(\alpha)$: $\begin{cases} [X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_5] = X_7,\ [X_2, X_7] = X_4, \\ [X_5, X_6] = X_4,\ [X_5, X_7] = \alpha X_4. \end{cases}$

$\mathfrak{n}_7^{118}$: $\begin{cases} [X_1, X_i] = X_{i+1},\ i = 2, 3, 5,\ \ [X_2, X_5] = X_7,\ [X_2, X_6] = X_4, \\ [X_3, X_5] = -X_4,\ [X_5, X_7] = -\frac{1}{4} X_4. \end{cases}$

— $c(\mathfrak{g}) = (3, 1, 1, 1)$.

$\mathfrak{n}_7^{119}$: $\begin{cases} [X_1, X_2] = X_3,\ [X_1, X_3] = X_4,\ [X_2, X_5] = X_4, \\ [X_6, X_7] = X_4. \end{cases}$

— $c(\mathfrak{g}) = (2, 2, 2, 1)$.

$\mathfrak{n}_7^{120}$: $[X_1, X_i] = X_{i+1},\ i = 2, 4, 6,\ \ [X_2, X_4] = X_7.$

$\mathfrak{n}_7^{121}$: $[X_1, X_i] = X_{i+1}.\ i = 2, 4, 6$

$\mathfrak{n}_7^{122}$: $[X_1, X_i] = X_{i+1},\ i = 2, 4, 6,\ \ [X_4, X_6] = X_7.$

$\mathfrak{n}_7^{123}$: $[X_1, X_i] = X_{i+1},\ i = 2, 4, 6,\ \ [X_2, X_4] = X_5,\ [X_4, X_6] = X_3.$

— $c(\mathfrak{g}) = (2, 2, 1, 1, 1)$.

$\mathfrak{n}_7^{124}$: $[X_1, X_i] = X_{i+1},\ i = 2, 4,\ \ [X_6, X_7] = X_5,\ [X_4, X_7] = X_3.$

$\mathfrak{n}_7^{134}$: $[X_1, X_i] = X_{i+1},\ i = 2, 4,\ \ [X_6, X_7] = X_5.$

— $c(\mathfrak{g}) = (2, 1, 1, 1, 1, 1)$.

$\mathfrak{n}_7^{126}$: $[X_1, X_2] = X_3,\ [X_4, X_5] = X_3,\ [X_6, X_7] = X_3.$

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